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Isospin Dependent Effective Interaction in Nucleon-Nucleus Scattering

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Abstract

The isospin-dependent component of the effective nucleon-nucleon interaction that causes the $\Delta T = 1$ (p, p') and (p, n) reactions off nuclei is studied. It is shown that the corrections to the impulse approximation comes from the g-matrix type correction and the rearrangement term. They are numerically estimated with the isospin-asymmetric nuclear-matter reaction matrix approach. The analysis of the isobaric analog transitions ^{42}Ca(p,n)^{42}Sc and ^{48}Ca(p,n)^{48}Sc are presented.

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The nucleon direct reaction of few hundred MeV to GeV projectile energy has become one of the preferred tools to extract the nuclear structure information complementing the reaction with the electromagnetic probes. Its success crucially depends on the good command of the effective nucleon-nucleon interaction used in the calculations based on the distorted-wave Born approximation (DWBA). From the theoretical point of view, there is a possibility of performing a convergent calculation of Watson expansion [1,2] or its variants [3], since the impulse-approximation is expected to be a reliable starting point at these energies. In practical analysis with the currently available computational resources, however, the treatment based on the density-dependent effective interaction [4,5] seems to be the only workable scheme to handle the medium corrections.

For the elastic scattering, the optical model potential obtained by folding the Brueckner reaction matrix (g-matrix) [6] is known to represent the leading terms in the Watson expansion [7], and the g-matrix approach has been reasonably successful in describing the elastic scattering experiments [4,5,8,9]. On the contrary, for the inelastic scattering, there had been no such general recipe. Many DWBA calculations had been performed, however, with the density dependent g-matrix without any theoretical justification. In this situation, we have shown for the simplest case, i.e. the isoscalar natural parity transitions, that the transition potential is the sum of the g-matrix and the rearrangement term which is written in terms of the derivative of the g-matrix with respect to the nuclear matter density [10,11]. Attempts to describe experimental data along this line have been pursued in recent years, and met a great success. The set of density-dependent effective interactions constructed by Kelly and collaborators [12-14] has been proven quantitatively accurate and practically useful to describe the elastic scattering and isoscalar natural parity excitations. In this background, we believe it quite urgent to identify medium corrections in other components of the effective nucleon-nucleon interaction than the spin-isospin independent part, since one of the advantage of the nucleon projectile over the electron is the wider variety of nuclear excitation modes accessible by it.

Although the isobaric analog excitation [15] has been one of the best-studied nuclear excitation modes, the isospin-dependent part of the effective interaction has been given very little theoretical attention. This unsatisfactory state of affairs

needs to be altered before any systematic empirical analysis be performed. In this Letter, we intend to identify the leading medium corrections to the isospin dependent component of the effective nucleon-nucleon interaction which induces the isovector nuclear excitations. In the formulation of such a process, the g-matrix in an asymmetric nuclear matter where the Fermi momentum of the proton differs from that of the neutron naturally appears. This g-matrix is used to evaluate the medium effects on the isovector transitions. We report the result of a numerical example at the incident energy $E_p = 150MeV$. It is applied to the isobaric analog transitions ^{42}Ca(p,n)^{42}Sc and ^{48}Ca(p,n)^{48}Sc at $E_p = 135MeV$. We hope that this work is to be the first step for the microscopic analysis of all the components of effective nucleon-nucleon interaction for intermediate energy nucleon scattering.

The g-matrix in an isospin-asymmetric nuclear matter [16] satisfies the Bethe-Goldstone equation formally identical to the one in the symmetric nuclear matter:

$$g = v + vG_0Qg \quad , \tag{1}$$

where v is the free nucleon-nucleon interaction, and G_0 and Q are the two-nucleon propagator and the Pauli operator in the medium, respectively. We neglect the self-energy term in G_0 assuming that the incident proton has sufficiently high energy. Then the difference of proton and neutron densities, ρ_p and ρ_n, generates *the isovector component* of Q, in addition to the isoscalar component which characterizes the isospin-symmetric nuclear matter. As a result, the effective interaction g of eq.(1) has four independent components g_{T,T_z} specified by the total isospin T and its z-component T_z of the two interacting nucleons. We write the g-matrix in the following way, featuring the tensor properties in isospin space:

$$g = g^{[0]} + g^{[\tau]}\tau_1 \cdot \tau_2 + g^{[\alpha]}(\tau_1 + \tau_2) + g^{[\beta]}\sqrt{2/3}[\tau_1 \times \tau_2]^{(2)} \quad , \tag{2}$$

where τ_1 and τ_2 are the isospin operators of two interacting nucleons. The four coefficients in the eq.(2) are written in terms of g_{T,T_z} as

$$g^{[0]} = \frac{1}{4}\left(g_{11} + g_{10} + g_{1-1} + g_{00}\right) \quad , \tag{3a}$$

$$g^{[\tau]} = \frac{1}{12}\left(g_{11} + g_{10} + g_{1-1} - 3g_{00}\right) \quad , \tag{3b}$$

$$g^{[\alpha]} = \frac{1}{4}\left(g_{11} - g_{1-1}\right) \quad , \tag{3c}$$

$$g^{[\beta]} = \frac{1}{6}\left(g_{11} - 2g_{10} + g_{1-1}\right) \quad . \tag{3d}$$

From eqs.(3a) to (3d), one recognizes that $g^{[0]}$, $g^{[\tau]}$ and $g^{[\beta]}$ are invariant with respect to the exchange of protons and neutrons in the medium, i.e. $\rho_p \leftrightarrow \rho_n$, while $g^{[\alpha]}$ changes its sign. These properties put restrictions of the possible form of density dependence of each term. A transparent view is obtained by defining the isoscalar density $\rho^{[s]} = \rho_p + \rho_n$, and the isovector density $\rho^{[v]} = \rho_p - \rho_n$. Specifically at the isospin-symmetric limit $\rho^{[v]} = 0$, one obtains

$$g^{[\alpha]} = g^{[\beta]} = \frac{\partial}{\partial \rho^{[v]}} g^{[0]} = \frac{\partial}{\partial \rho^{[v]}} g^{[\tau]} = \frac{\partial}{\partial \rho^{[v]}} g^{[\beta]} = 0 \quad . \tag{4a}$$

Also, taking the leading density-dependence of $g^{[\alpha]}$, one obtains

$$g^{[\alpha]} \approx \rho^{[v]} \frac{\partial}{\partial \rho^{[v]}} g^{[\alpha]} \quad . \tag{4b}$$

We now consider the elastic and inelastic scatterings in order. The elastic optical model potential U of a nucleon in the isospin asymmetric nuclear matter is given by folding the effective interaction g with the asymmetric nuclear density. It is convenient to split the resultant optical model potential into isoscalar and isovector part *á la* Lane as [17]

$$U = U^{[s]} + U^{[v]}\,\tau_z \quad , \tag{5}$$

where each term is given by

$$\begin{aligned} U^{[s]} &= g^{[0]}\rho^{[s]} + g^{[\alpha]}\rho^{[v]} \\ &\approx g^{[0]}\rho^{[s]} \quad , \end{aligned} \tag{6a}$$

and

$$
\begin{aligned}
U^{[v]} &= g^{[\alpha]}\rho^{[s]} + \left(g^{[\tau]} + g^{[\beta]}\right)\rho^{[v]} \\
&\approx g^{[\alpha]}\rho^{[s]} + g^{[\tau]}\rho^{[v]} \\
&\approx \left(\rho^{[s]}\frac{\partial}{\partial\rho^{[v]}}g^{[\alpha]} + g^{[\tau]}\right)\rho^{[v]} \quad .
\end{aligned}
\tag{6b}
$$

In the derivation of $U^{[s]}$ and $U^{[v]}$, we have dropped all terms that are of second or higher orders in powers of $\rho^{[v]}$. In the last line of eq.(6b), the relation (4b) is used. The above expression shows that the optical model potential U of eq.(5) is obtained by folding

$$
\begin{aligned}
v_{el} &= g^{[0]}\rho^{[s]} + \big(\rho^{[s]}\frac{\partial}{\partial\rho^{[v]}}g^{[\alpha]} + g^{[\tau]}\big)\tau_1\cdot\tau_2 \\
&\equiv v_{el}^{0} + v_{el}^{\tau}\ \tau_1\cdot\tau_2 \quad ,
\end{aligned}
\tag{7}
$$

with the nuclear densities $\rho^{[s]}$ and $\rho^{[v]}$. One can regard v_{el} *defined by* eq.(7) as the effective interaction for the elastic scattering.

For the inelastic scatterings, both through the diagrammatical analysis of Watson expansion and the macroscopic collective excitation model, one can relate the inelastic transition potential and the density derivative of the optical potential U for the elastic scattering [10,11]. Assuming that the transition of the target nucleus is described by the isoscalar and isovector transition densities $\rho_{tr}^{[v]}$ and $\rho_{tr}^{[v]}$, one can write the transition potential U_{tr} which describe the inelastic scattering as

$$
U_{tr} = \rho_{tr}^{[s]}\frac{\partial}{\partial\rho^{[s]}}U^{[s]} + \rho_{tr}^{[v]}\frac{\partial}{\partial\rho^{[v]}}U^{[v]}\,\tau_z \quad .
\tag{8}
$$

This yields, for $N = Z$ nuclei ($\rho^{[v]} = 0$), the expression

$$
U_{tr} = \rho_{tr}^{[s]}(g^{[0]} + \rho^{[s]}\frac{\partial}{\partial\rho^{[s]}}g^{[0]}) + \rho_{tr}^{[v]}(g^{[\tau]} + \rho^{[s]}\frac{\partial}{\partial\rho^{[v]}}g^{[\alpha]})\tau_z \quad ,
\tag{9}
$$

which explictly shows that the transition potential can be obtained from the effective interaction

$$
v_{in} = v_{in}^{[0]} + v_{in}^{[\tau]}\tau_1\cdot\tau_2 \quad ,
\tag{10}
$$

where the isospin-independent and dependent components, $v_{in}^{[0]}$ and $v_{in}^{[\tau]}$, are given by

$$v_{in}^{[0]} = g^{[0]} + \rho^{[s]} \frac{\partial}{\partial \rho^{[s]}} g^{[0]} \quad , \tag{11a}$$

and

$$v_{in}^{[\tau]} = g^{[\tau]} \ + \rho^{[s]} \frac{\partial}{\partial \rho^{[v]}} g^{[\alpha]} \quad . \tag{11b}$$

The second term of eq.(11a) is the so-called rearrangement term for the isoscalar transition [10] which roughly doubles the density dependence of the inelastic effective interaction. Eq.(11b) is one of the main result of this Letter. Its second term is analogous to the one in eq.(11a), and no less remarkable since it persists at $\rho^{[v]} = 0$ despite that $g^{[\alpha]}$ itself vanishes at this limit. We now compare eq.(7) with eq.(11), i.e. the effective interactions for elastic and inelastic scatterings. For the isospin-independent component, one recognizes

$$v_{in}^{[0]} = \left(1 + \rho^{[s]} \frac{\partial}{\partial \rho^{[s]}} \right) v_{el}^{[0]} \quad , \tag{12}$$

which has been instrumental in determining the empirical density-dependent interaction [12-14]. For the isospin-dependent component, one finds

$$v_{in}^{[\tau]} = v_{el}^{[\tau]} = g^{[\tau]} \ + \rho^{[s]} \frac{\partial}{\partial \rho^{[v]}} g^{[\alpha]} \quad . \tag{13}$$

This shows that the extra factor appearing in eq.(12) can be formally eliminated in the case of isospin-dependent component. We stress that this *does not* mean the absence of the rearrangement term. It is achieved by natural but purely formal redefinition, eq.(7). In principle, therefore, with eq.(13), the isospin-dependent empirical interactions can be constructed from the combined analysis of the elastic scattering and the $\Delta T = 1$ inelastic and charge-exchange reactions on $N \sim Z$ nuclei.

We now turn to the numerical assessment of the medium correction to the isospin-dependent component of the effective interaction. The detailed description of the technique to solve eq.(1) in an isospin-asymmetric nuclear matter and the full results will be published elsewhere. Here, we show the relevant result of an

example of such calculation. At the projectile energy of $E_p = 150 MeV$, Reid Soft Core potential as the input free two nucleon potential, we obtain the following number for the volume integrals of isospin dependent components of g-matrix.

$$g^{[\tau]} = [\{22 + 75i\} + \{-24 - 53i\}\rho^{[s]}] + [\{8 + 22i\} + \{17 - 17i\}\rho^{[s]}]\mathbf{L}\cdot\mathbf{S} \ , \qquad (14)$$

$$g^{[\alpha]} = [\{6 + 38i\}\rho^{[v]}] + [\{-1 + 1i\}\rho^{[v]}]\mathbf{L}\cdot\mathbf{S} \quad , \qquad (15)$$

where the densities $\rho^{[s]}$ and $\rho^{[v]}$ are normalized to the nuclear matter saturation density. The isospin dependent component of the g-matrix, $g^{[\tau]}$, is identical to the one in the symmetric nuclear matter in ref.[18]. It includes the density dependence induced by the nonlocality. Only the contribution from the allowed channel is included, and thereby the antisymmetrization is taken into account. The numbers in eqs.(14) and (15) can be also interpreted as the strength of the interaction in zero-range approxiamation. One observes that, at this energy, the density dependence of $g^{[\alpha]}$ tends to cancel that of $g^{[\tau]}$, which already exists in the g-matrix in the symmetric nuclear matter, in the calculation of the isospin-dependent effective interaction, eq.(13), especially in the imaginary part of the central force. More to the point, these two density-dependent corrections are of the same order numerically. It clearly shows that the partial inclusion of the medium correction, only with g-matrix, for example, cannot be very meaningful.

In order to show how the density-dependent effective interaction eqs.(13-14) works in the inelastic scattering, we show the result of DWBA calculations [19] for the isobaric analog transitions ^{42}Ca(p,n)^{42}Sc and ^{48}Ca(p,n)^{48}Sc at $E_p = 135 MeV$. For the facility of the calculation, we adopt following two approximations: First, we take the empirical optical model potential by Schwandt *et al* [20], instead of calculating it microscopically. Second, we replace the density-independent part of eqs.(14) and (15) by the three-range Yukawa parametrization of free two-nucleon scattering matrix by Franey and Love [21] at $E_p = 140 MeV$ (FL140). The density-dependent portion of the above effective interactions is treated as Yukawa force of very short range. The calculated differential cross section is shown in Figures, where dashed line corresponds to the impulse approximation with FL140 interaction; dotted-dashed line, the result with $v_{in}^{[\tau]}$ of eq.(11b) being replaced by $g^{[\tau]}$;

solid line, the result with the full medium correction included. We can see the following effect: First, the calculation with the t-matrix overestimates the experimental cross section [22]. Second, by changing the interaction to the g-matrix, the calculation lowers the cross section, but overshoot the data. Last, by including all the medium effects, the calculation hits the experimental data points very well. A warning is due, however, to the overemphasis on the favourable comparison with the experiment at this point, since the uncertainty of the isospin-dependent component of the free t-matrix is already very large. Also, the approximations in the current calculation leaves some space for further improvement. Rather, we would reiterate our basic point that the medium modification should be, and can be estimated with the proper inclusion of the rearrangement term.

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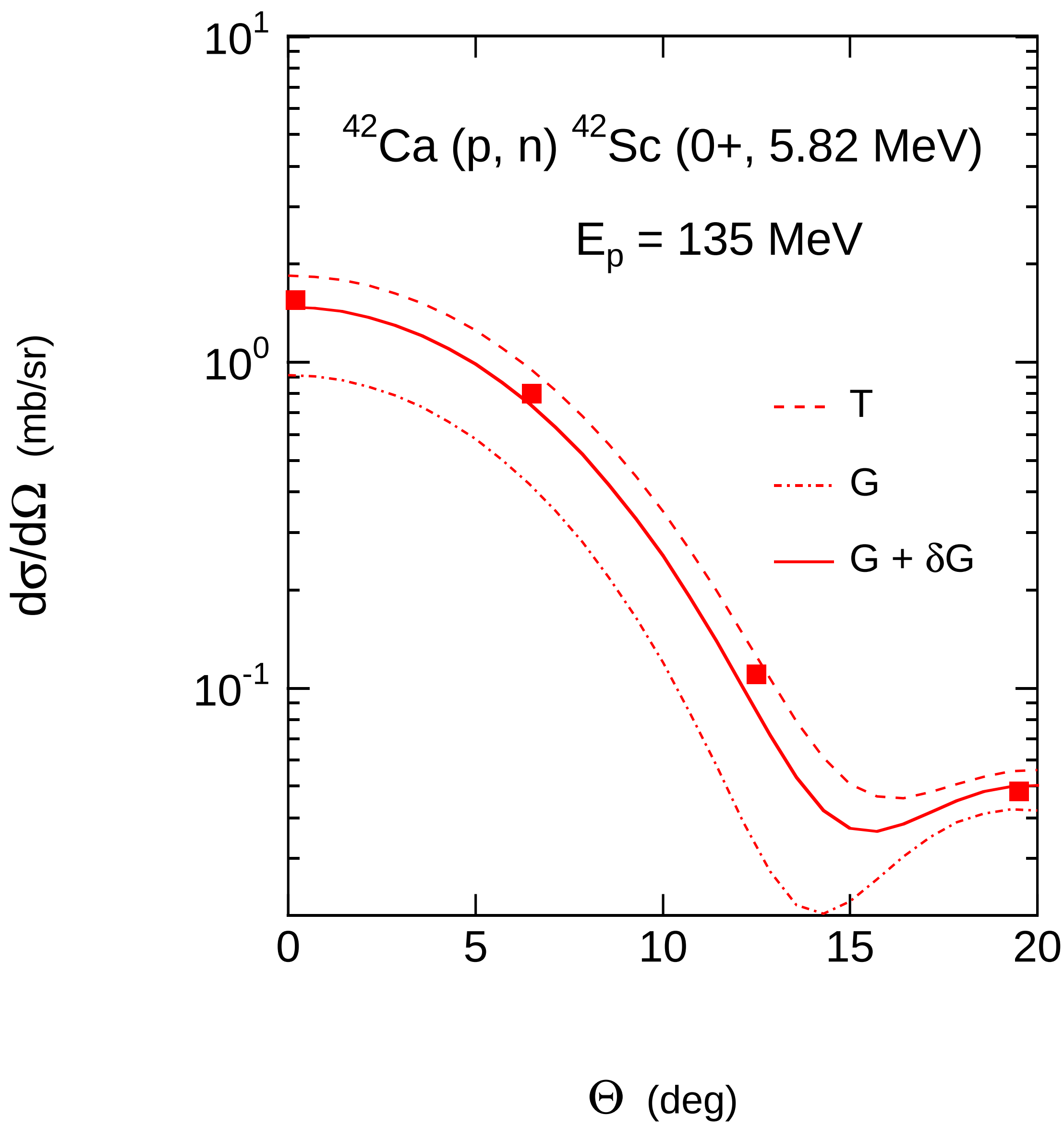
Figure Captions

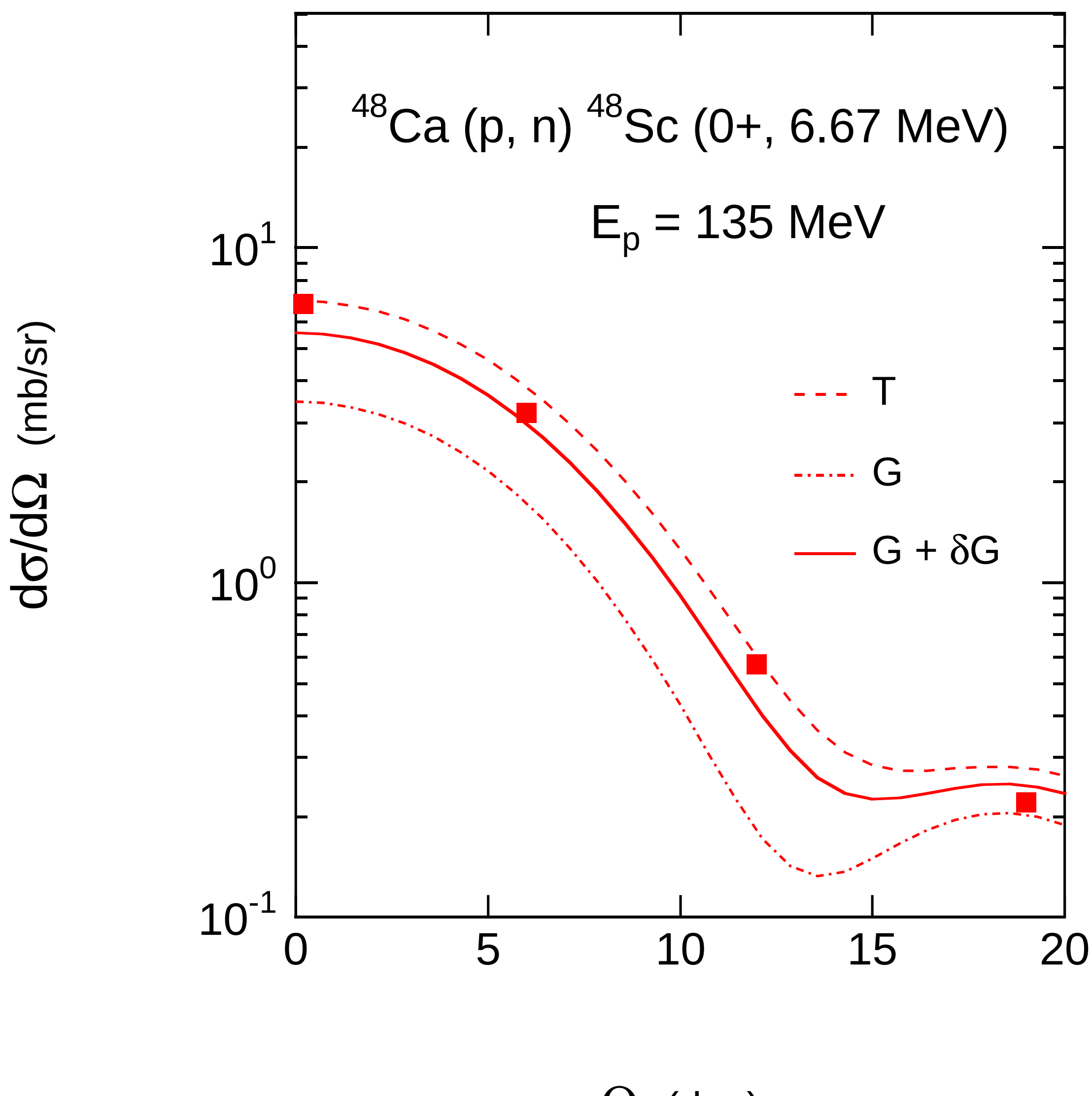
Fig. [I]: The differential cross section for the reaction ^{42}Ca(p,n)^{42}Sc. The dashed line is the result of the impulse approximation. The dotted-dashed curve partially accounts for the medium corrections, while the solid line includes all the medium modifications.

Fig. [II]: The differential cross section for the reaction ^{48}Ca(p,n)^{48}Sc. Notation is the same as for fig.[I].



42Ca (p, n) 42Sc (0+, 5.82 MeV)
Ep = 135 MeV
T
G
G + δG
dσ/dΩ (mb/sr)
Θ (deg)
10^1
10^0
10^-1
0
5
10
15
20

Fig. 1



^{48}Ca (p, n) ^{48}Sc (0+, 6.67 MeV)
E_p = 135 MeV
T
G
G + δG
10^{1}
10^{0}
10^{-1}
0
5
10
15
20
dσ/dΩ (mb/sr)
Θ (deg)

Fig. 2